MATH 2B Prep: Sigma Notation

Facts to Know:

Sigma notation is used to write down long sums in a concise form.

Definition:

- $a_1 + a_2 + \cdots + a_{10} = \sum_{i=1}^{10} a_i$
- $a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i$
- $a_1 + a_2 + \cdots = \sum_{i=1}^{\infty} a_i$

Examples:

1. Write $1 + 3 + 5 + \cdots + 99 + 101$ and $\frac{1}{n} + \frac{2}{n} + \cdots + \frac{n}{n}$ in sigma notation.

$+2 \quad +2 \quad +2$

$1 = 2(1) - 1$

$3 = 2(2) - 1$

$5 = 2(3) - 1$

$\vdots$

$101 = 2(51) - 1$

$a_i = 2i - 1$

$$\boxed{\sum_{i=1}^{51} 2i - 1}$$

$\frac{1}{n} + \frac{2}{n} + \cdots + \frac{n}{n}$

$+\frac{1}{n} \quad +\frac{1}{n}$

$\frac{1}{n} = 1 \cdot \frac{1}{n}$

$\frac{2}{n} = 2 \cdot \frac{1}{n}$

$\vdots$

$\frac{n}{n} = n \cdot \frac{1}{n}$

$a_i = \frac{i}{n} = i \cdot \frac{1}{n}$

$$\boxed{\sum_{i=1}^{n} \frac{i}{n}}$$

2. Rewrite $\sum_{j=0}^{n-1} \sin\left(\frac{j+1}{n}\right)$ as a sum starting at 1 instead of zero. (This is called a change/shift of index)

$$\sum_{j=0}^{n-1} \sin\left(\frac{j+1}{n}\right) = \underset{i=1}{\sin\left(\frac{1}{n}\right)} + \underset{i=2}{\sin\left(\frac{2}{n}\right)} + \cdots + \underset{i=n}{\sin\left(\frac{n}{n}\right)}$$

$a_i = \sin\left(\frac{i}{n}\right)$

$$\boxed{\sum_{i=1}^{n} \sin\left(\frac{i}{n}\right)}$$

3. Simplify the expression $\sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+2}$. What happens as n goes to infinity?

$$\underbrace{\sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+2}}_{a_i} = \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \cdots + \left(\cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+2}\right)$$

$$= \boxed{1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}}$$

$$\lim_{n\to\infty} 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} = 1 + \frac{1}{2} + 0 + 0 = \boxed{\frac{3}{2}}$$